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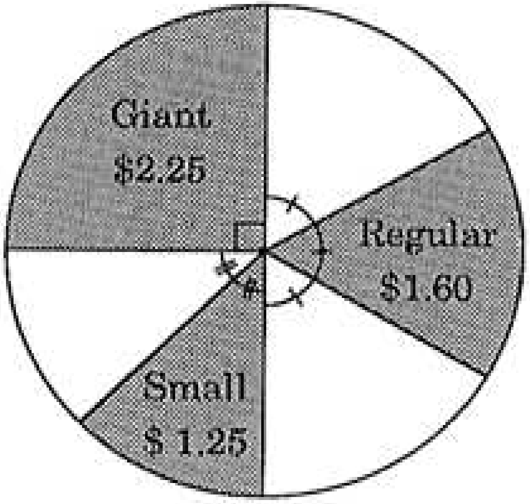

Champagne's Pizzeria

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Impact of Mathematics Content Courses on Elementary Preservice Teachers' Confidence in Teaching Mathematics

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Abstract

There is a general consensus that mathematical content knowledge (M-CK) is crucial for enabling elementary school teachers to effectively teach mathematics. However, it has been suggested that M-CK is not sufficient for elementary school teachers – it must be accompanied by math pedagogical content knowledge (M-PCK). In order to better identify coursework that may promote M-CK and M-PCK, this study investigates confidence of M-CK and M-PCK of elementary preservice teachers (PSTs) who have participated in math content coursework designed for elementary teachers. Findings from preliminary work in a larger study suggest that PSTs who take one or more of these content courses, along with a mathematics methods course, have greater confidence related to M-CK and M-PCK than PSTs who take only traditional mathematics courses along with a mathematics methods course. The research is ongoing.

Current research suggests the importance of mathematical content knowledge to enable elementary school teachers to teach mathematics effectively (Ball, 2003; Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992; NCTM, 2003). However, there is not a consensus on how best to prepare elementary preservice teachers (PSTs) in order to achieve mathematical content knowledge (M-CK) that translates to effective mathematical pedagogical content knowledge (M-PCK) and, in turn, student learning (Kirtman, 2008). For example, Ball (2003) notes that "increasing the quantity of teachers' mathematics coursework will only improve the quality of mathematics teaching if teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work. The goal is not to produce teachers who know more mathematics. The goal is to improve students' learning" (p. 1). In other words, more content area mathematics courses, while they may support increased M-CK, may not translate to M-PCK.

Investigating confidence of M-CK and M-PCK of PSTs related to specific course experiences during a teacher preparation program may allow researchers and teacher educators to better identify course experiences that may enhance M-CK and M-PCK of PSTs. This research investigated influences of mathematics coursework that has been *designed specifically with elementary PSTs in mind*—mathematics content courses taught in the Math Department, but with M-PCK as an emphasis. Confidence with respect to M-CK and M-PCK was investigated – comparing PSTs who participated in the specific math content courses with those who did not participate.

Perspectives

Research connected to self-efficacy (Bandura, 1986) and teacher efficacy (Henson, 2001; Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998) provides a framework for this study. Self-efficacy can be described as belief that one is capable of accomplishing certain tasks or goals (Bandura, 1986). Teacher efficacy relates to the teacher's belief in his/her capability to accomplish particular teaching-related tasks or goals (Tschannen-Moran et al., 1998). Teacher efficacy has been found to influence teachers' persistence and willingness to try new ideas; additionally, it has been linked to student outcomes such as achievement (Henson, 2001; Tschannen-Moran et al., 1998). Experts in instrument development, such as Gable and Wolf (1993), provide examples showing that self efficacy can be measured by rating confidence of specific beliefs about behaviors. This suggests that an instrument that measures confidence levels with respect to M-CK and M-PCK should provide indicators of self-efficacy with respect to these constructs. Indeed, NCTM (2003) noted, "Candidates' comfort with, and confidence in, their knowledge of mathematics affects both what they teach and how they teach it" (p. 4). Thus, assessing changes in PSTs' confidence toward M-CK and M-PCK may provide indicators of self-efficacy that may impact future mathematics teaching practices.

In order to measure confidence with respect to M-CK and M-PCK, we investigated instruments that have been used extensively and found to be trustworthy. The Fennema-Sherman Mathematics Attitude Scale (FSMAS) has been used for more than 20 years to investigate attitudes towards mathematics (Mulhern & Rae, 1998), providing a solid base from which to build an instrument to measure PSTs' confidence related to M-CK and M-PCK.

In addition to confidence, it is important to consider other indicators of M-CK and M-PCK. For example, Deborah Ball noted in remarks to the Secretary's Summit on Mathematics (2003) that teaching mathematics effectively in elementary schools requires that "teachers must know the same things that we would want any educated member of society to know, but *much more*" (p. 7). The "much more" (M-PCK) entails being able to ask and answer *why* about mathematical problems; fluency with and ability to strategically use representations; ability to inspect and make sense of and use students' mathematical methods; capacity to support mathematical language, and *much more*. Therefore, in addition to confidence items, we designed open-ended problems for the PSTs to complete that related to their own M-CK (solving and explaining mathematical problems) and their M-PCK (examining and commenting on non-routine student work). These provide additional data related to PSTs' M-CK and M-PCK that are aligned with ideas described by Ball (2003).

In Ball's remarks to the Secretary's Summit, she noted that few mathematics courses offer opportunities that would produce knowledge that is appropriate for elementary school teachers. Further, she urged that "ongoing research in this area is crucial" (p. 9). With these important issues in mind, we asked: Does elementary preservice teachers' confidence in M-CK and M-PCK change before and after completing the math methods course? If so, do the math content courses influence this change? In particular, in this article we explore the following research questions:

1. How does the change in confidence towards mathematical content knowledge (M-CK) before and after completion of a mathematics methods course compare between students who take math content courses designed for elementary school teachers and those who do not take these courses?
2. How does the change in confidence towards math pedagogical content knowledge (M-PCK) before and after completion of a mathematics methods course compare between students who take math content courses designed for elementary school teachers and those who do not take these courses?

An earlier paper (Truxaw, Cardetti & Bushey, 2010) reported on the collaborative process across the disciplines of mathematics and mathematics education that initiated these questions. This article builds from that work, focusing on the research questions themselves rather than the collaborative process.

Methods

Data for this paper are drawn from a larger research study that uses mixed methods (Creswell, 1998; Johnson, & Onwuegbuzie, 2004) to investigate influences of math content courses geared toward elementary PSTs. In order to examine confidence related to M-CK and M-PCK, a survey was administered that includes Likert items adapted from the *Fennema-Sherman Mathematics Attitude Scale* (Mulhern & Rae, 1998), along with open-ended content problems designed to uncover both M-CK and M-PCK. Additionally, the larger study includes interviews with a subset of the PSTs to provide supporting data related to the influence of the content courses.

Context

Participants are elementary PSTs enrolled in the teacher preparation program (TPP) at a large public research university in the northeastern United States. For the larger study, participants include elementary education PSTs in their junior and senior years in the TPP. These students are predominantly female (90-95%), white (80-90%), and typical ages range from 20 to 25 years old. For this paper, we focus on participants during the fall of their senior year, prior to and after completion of the required math methods course.

At our institution, all elementary education PSTs are required to take a mathematics methods course within the School of Education, along with at least three “quantitative” content courses (e.g., mathematic or statistics) outside the School of Education. The Department of Mathematics offers two content courses (recommended but not required by the TPP) for elementary PSTs. The courses have been created to develop an advanced perspective on and profound understanding of concepts, structures, and algorithms constituting the core of K-8 math curriculum. The topics of the course are chosen to support and extend the expectations set forth by the Mathematical Standards, K-8 (NCTM, 2000). The class meetings are structured to provide students with the experience of developing their own mathematical ideas. The instructor acts as a facilitator providing guidance to lead students toward understanding of concepts behind familiar concepts as well as new ones. Special attention is given to exploring and communicating the ideas and reasons behind the mathematical manipulations. Participants who completed either of these courses, along with the math methods course, are referred to here as the C-group (content). The participants who completed the math methods course, but did not complete either of the content courses recommended for elementary PSTs are referred to here as the NC-group (non-content).

Data Collection and Analysis

Data collection included pre- and post-surveys administered to the elementary PSTs in the math methods courses in fall 2009. Surveys included demographic items; Likert-type items to measure confidence in M-CK and M-PCK; and open-ended response items to gauge PSTs' facility to accurately complete and explain content problems, as well as to interpret and explain student work samples. Overall, 23 PSTs (11 C, 12 NC) completed the pre-survey, 32 PSTs completed the post-survey (16 C, 16 NC), and 19 PSTs (9 C, 10 NC) completed both pre- and post-surveys. Analysis focuses on the 19 participants who completed both pre- and post-surveys. It is important to note that the pre and post designations relate to the *methods course*, not the content course. Related to the research questions, we wondered if the C-group would begin the methods course with different levels of confidence in M-CK and/or M-PCK than the NC-group; *and* we wondered if the C-group would demonstrate different levels of *change in confidence* in M-CK and/or M-PCK than the NC-group (from pre to post methods course). In other words, we were interested in the impact of the specific content courses on confidence relative to the methods course.

For this paper we focus on the Likert-scale scores (1=Strongly Disagree; 5=Strongly Agree). The Likert-scale items were organized according to confidence in mathematical content (M-CK, items 1-4) and confidence in teaching mathematics (M-PCK, items, 5-8), as follows:

1. Generally, I feel secure about attempting mathematics.
2. I have a lot of self-confidence when it comes to math.
3. Mathematics is enjoyable and stimulating to me.
4. I would rather figure out a math problem myself than to have someone give me the solution.

5. Generally, I feel secure about teaching elementary school mathematics.
6. I have a lot of self-confidence when it comes to teaching elementary school mathematics.
7. Teaching elementary school mathematics is enjoyable and stimulating to me.
8. I would rather if my elementary school student could figure out a math problem rather than having me give them the solution.

Although items 4 and 8 relate to M-CK and M-PCK respectively, they deal less with overall confidence and more with a stance toward problem solving. Therefore, when items were aggregated for certain analyses, items 1-3 and items 5-7 were grouped and items 4 and 8 were considered independently.

A preliminary analysis focusing *only* on the change in mean differences of the Likert-scale scores was presented in Truxaw, Cardetti & Bushey (2010). In this study, we analyze the results of paired *t* tests (significant at the .05 level) using SPSS software (Green, Salkind, & Akey, 2000) comparing these mean differences (pre-score minus post-score – a negative difference indicating positive change). Due to the small sample size, we report a 95% confidence interval (CI) of the paired *t* test analysis rather than the *p*-values. If zero did not fall within the range of a 95% CI, it indicated 95% confidence that the difference between the pre- and post-survey means was not zero and, therefore, the mean difference was significant (Shavelson, 1996).

As the larger study progresses, results are being triangulated through analysis of the open-ended content problems and interviews. The open-ended content problems are being scored using rubrics that have been developed, refined, and tested for inter-rater reliability (90% or higher). The interview data is being coded and analyzed thematically (e.g., Creswell, 1998; Strauss & Corbin, 1990). This paper reports on findings from the Likert-items, along with some preliminary findings from the open-ended items.

Results and Discussion

In this section we present results of analysis used to answer the research questions (RQs) along with related explanations and discussion. We present descriptive statistics, mean differences, and the 95% CI for a mean difference for the corresponding Likert item scores, along with explanations and discussion of these results. Our analysis and discussion focus on the statistical significance of each individual item, as well as on the aggregated data for groups of items corresponding to different constructs.

RQ 1: Comparison of elementary PSTs' Confidence in Mathematical Content Knowledge

To answer the first RQ, data from pre- and post-scores of the Likert items related to M-CK (items 1-4) were analyzed. Table 1 summarizes the changes in pre- and post-scores for each of the M-CK-related items for the C- and NC-groups.

Table 1

Confidence Toward Math Content

	Pre		Post		*Mean Difference pre-post*	*95% CI Mean Difference*
	Mean	*SD*	*Mean*	*SD*		
			Content Group, *n = 9*			
Item 1	4.44	.53	4.33	.50	.11	(-1.00, .20)
Item 2	4.44	.73	4.00	.50	**.44**	**(.04, .85)****
Item 3	4.33	1.00	4.22	.44	.11	(-.60, .824)
Item 4	4.22	.67	4.44	.53	-.22	(-.86, .42)
			Non-Content Group, *n=10*			
Item 1	3.40	.97	3.80	.63	-.40	(-1.00, .20)
Item 2	3.10	.87	3.40	.84	-.30	(-.78, .18)
Item 3	3.20	.79	3.50	.71	-.30	(-.65, .05)
Item 4	4.00	.47	4.00	.47	.00	

**Significant at the 95% CI level

Before making pre and post comparisons, it should be noted that mean scores for *every item* related to M-CK, both pre and post, were higher for the C-group than for the NC-group. In terms of pre and post comparisons, for the C-group, only one item (item 2 - *I have a lot of self-confidence when it comes to math*) showed significant change at the 95% level from pre-to post-survey (*95% CI*, (.04, .85)). A large effect size of .72 was found for this item when comparing pre- and post-survey scores. Interestingly, the change was a *negative* one – that is, the C-group reported significantly *less confidence* for this item than prior to participating in the methods course. Although not significant, mean scores for items 1 and 3 decreased from pre to post; for item 4, mean scores increased from pre to post. For the NC-group, although no single item showed significant changes, the mean scores showed *increases* in reported confidence for three out of the four content-related items (items 1-3) and *no change* for item 4.

There were interesting, although not statistically significant, observations of the data related to item 4 (*I would rather figure out a math problem myself than to have someone give me the solution*). While the mean scores for the C-group on item 4 increased (pre, M=4.22; post, M=4.44), these were not significantly different at the 95% CI level. For the NC group there were no changes for this item in means or standard deviations (pre and post, M=4.00, SD=0.471). However, these results show that both groups reported consistently high expectations for allowing themselves to struggle when working through mathematical problems, with the C-group reporting slightly higher levels and slight increase from pre- to post. These results will be discussed further in comparison to a similar item (item 8) related to M-PCK.

Additionally, to further understand changes in pre- and post-scores for each group related to this RQ, three Likert items constituting a single construct, confidence toward M-CK, were identified (items 1-3). The aggregate scores' mean differences and CIs were analyzed for the two groups separately. The analysis yielded no significant change at the 95% level for the C-group

(Mean difference, .22; *95% CI*, (-.12, .56)). However, a significant (positive) change was noted for the NC-group (Mean difference, -.33; *95% CI*, (-.65, -.02)). A medium effect size of .48 was found for this construct when comparing pre- and post-survey scores. These results suggest that the NC-group of PSTs showed significant growth in confidence toward M-CK after completing the math methods course. One may conjecture that NC-group's lack of exposure to the M-CK courses may have impacted their need to focus on the mathematical content while in the methods course.

RQ 2: Comparison of elementary PSTs' Confidence in Mathematical Teaching

To address the second research question, data from the pre- and post-scores of the Likert items related to M-PCK (items 5-8) were analyzed. Table 2 summarizes the changes in pre- and post-scores for each of the M-PCK related items for the C- and NC-groups.

Table 2

Confidence Toward Teaching Math

	Pre		Post		*Mean Difference pre-post*	*95% CI Mean Difference*
	Mean	*SD*	*Mean*	*SD*		
Content Group, *n=9*						
Item 5	3.33	.50	3.78	.67	**-.45**	**(-.85, -.04)****
Item 6	3.22	.67	3.78	.67	**-.56**	**(-.96, -.15)****
Item 7	3.67	.87	4.00	.71	-.33	(-.72, .42)
Item 8	4.89	.33	4.89	.33	.00	
Non-Content Group, *n=10*						
Item 5	3.70	.48	3.20	.79	.50	(-.11, 1.11)
Item 6	3.10	.57	2.80	.79	.30	(-.46, 1.06)
Item 7	3.30	.48	3.10	.74	.20	(-.46, .86)
Item 8	4.50	.53	4.50	.71	.00	(-.48, .48)

**Significant at the 95% CI level

Before reporting results of the CI analysis, it is worth noting general trends related to confidence toward teaching mathematics. Although the results were less definitive than for M-CK, the mean scores related to M-PCK suggest greater confidence by the C-group than the NC-group. For example, prior to taking the math methods course, the C-group reported greater confidence than the NC-group on three of the four items (items 6, 7 & 8). After completing the math methods course, the C-group reported greater confidence than the NC-group *on every item* related to M-PCK.

For the C-group, the analyses of the CIs for mean differences showed significant increases at the 95% level from pre- to post-survey for items 5 (*95% CI*, -.85, -.04) and 6 (*95% CI*, (-.96, -.15)). Large effect sizes of 0.88 and 0.84, respectively, were found when comparing pre- and post-survey scores for these items. For item 7, the C-group showed an increase in mean scores, but the change was not significant at the 95% level; for item 8, there was no change in mean scores or standard deviations (pre and post, *M*=4.89, *SD*=0.333). For the NC-group, although there were decreased mean scores on three of the four items (items 5, 6 & 7), none of the changes were significant. For item 8, the NC-group showed no change from pre- to post-survey in the mean scores (pre, *M*=4.50, *SD*=.527; post, *M*=4.50, *SD*=.707). These results show that both groups reported consistently high expectations for allowing their students to struggle when working through mathematical problems, with the C-group reporting slightly higher levels.

Additionally, to further understand changes in pre- and post-survey scores for each group related to M-PCK, three Likert items constituting a single construct: confidence toward M-PCK were identified (items 5-7). The confidence intervals for aggregate means differences on pre- and post-scores for this construct were analyzed for the two groups separately. The analysis yielded a *positive* significant change at the 95% level for the C-group (*Mean difference*, -.44; *95% CI*, (-.76, -.13)). A moderately large effect size of .68 was found for this construct when comparing pre- and post-survey scores. However, no significant change was noted at the 95% level for the NC-group (*Mean difference*, .33; *95% CI*, (-.24, .91)). These results suggest that the C-group of PSTs showed significant growth in confidence toward M-PCK after completing the math methods course. One may conjecture that the C-group of PSTs' prior exposure to M-CK courses may have provided them content knowledge that allowed them to focus on the pedagogical content while in the methods courses, thus increasing their confidence with teaching mathematics while participating in the math methods course.

There were interesting observations of the data related to item 8 (*I would rather if my elementary school student could figure out a math problem rather than having me give them the solution*). This question was designed to parallel item 4 in the M-CK section (related to a similar stance toward their own problem-solving). As noted, neither group of PSTs reported changes related to item 8. Recall that for the NC-group there were no changes in mean scores for item 4 either – that is, the NC-group reported no change in stance toward problem-solving for themselves or for their students. For the C-group, there was a slight increase in expectations related to their own problem solving, but no increase in expectations toward preferences for their students' problem solving. It should be noted that both groups reported high expectations for both themselves and their students related to persistence with problem solving. Even so, it is interesting to observe that, for both C- and NC-groups, the mean scores for item 4 (expectations for self) were *lower* than the mean scores for item 8 (expectations for students) – suggesting that the PSTs may have different mathematical expectations for themselves than for their students.

Preliminary Findings from Open-Ended Content Items

Results from open-ended content items are being used to corroborate findings from the survey related to PSTs' M-CK and M-PCK. Analysis of these items is in progress. They are being scored using rubrics that have been developed, refined, and tested for inter-rater reliability (90% or higher). Preliminary results from analysis of a subset of the larger data set suggest that the C-group, as compared to the NC-group, of PSTs demonstrates stronger content knowledge for doing math, interpreting student work, attempting explanations, and providing reasonably accurate/appropriate explanations. This ongoing analysis will further support this research.

Limitations

One limitation of this study is its small sample size. However, the results demonstrate significant means difference at the 95% confidence level. This suggests that if hypothesis testing were performed that the results would likely reach statistical significance. We expect to confirm this statement as we continue to collect and analyze data over time. Another limitation may lie in the particular TPP where these data are collected. More research studies, both incorporating more participants and within other TPPs, could provide further evidence of the extent to which the findings reported here are representative of the experiences of other PSTs in other TPPs. Finally, this research investigates *confidence* rather than knowledge or observed teaching practices; while confidence provides a window into potential knowledge and practice, we recognize that there are limitations in using this approach. Future research investigating a link between confidence and performance will continue to grow the body of literature on preservice elementary preservice teachers' mathematics and pedagogical content knowledge.

Final Remarks

This study sought to uncover influences that mathematics content courses designed for elementary PSTs may have on the PSTs' confidence toward M-CK and M-PCK. The findings indicate that the C-group showed greater gains than the NC-group in confidence towards M-PCK after taking the required math methods course. Interestingly, students in the NC-group reported greater gains than the C-group in confidence towards M-CK. These results suggest that PSTs going into the methods courses with greater M-CK may potentially increase their confidence in M-PCK, and that this increase could be to a greater extent than those with less M-CK experience. The NC-group's gains in M-CK and relative lack of gain in M-PCK from pre- to post-methods course may suggest that their attention was focused more on the mathematics content than the teaching methods or student learning.

An interesting additional result was that while both C- and NC-groups reported high expectations for allowing their students to struggle when working through mathematical problems, the expectations they reported for themselves, although also relatively high, were lower than for their students. In essence, they reported lower tolerance for personal persistence with problem solving for themselves than for their students.

Based on these preliminary findings, we are continuing to analyze the data and to expand the study to include: further analysis of open-ended items, administration of surveys to additional PSTs, and interviewing of selected PSTs. The analysis of the interviews will allow us to better interpret survey responses and to dig deeper into their thinking and perceptions related to M-CK and M-PCK.

Our findings corroborate others (e.g., Ball, 2003; Ball, Lubienski, & Mewborn, 2001; NCTM, 2003) who have noted the importance of both M-CK *and* M-PCK for elementary school teachers. Our results suggest that one means of supporting elementary PSTs as they work to become effective mathematics teachers is participation in mathematics content courses that are designed specifically with elementary school teachers in mind. Indeed, these mathematical *content* courses may enhance learning outcomes of mathematics *methods* courses by providing sufficient M-CK to allow the PSTs to focus their attention, during *methods* courses, on the teaching methods and student learning related to the mathematics. Without these courses, the PSTs' attention toward student learning and mathematical teaching methods may be diluted while they are focusing on their own mathematical content knowledge. It will be important for future investigations to continue to investigate the influences of such courses on M-CK and M-PCK.

Endnotes

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Using Language Objectives to Support Linguistically Diverse Students in Mathematics Classes

by Megan Staples – University of Connecticut and
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Language is critical to the teaching and learning of mathematics for all students (Chapin, O'Connor, & Anderson, 2003; Lampert & Blunk, 1998; National Council of Teacher of mathematics (NCTM), 2000; Pimm, 1987) and for the growing population of English language learners (ELLs) in particular (Brenner, 1998; Fernandes, Anhalt, & Civil, 2009; Janzen, 2008; Lee & Woo, 2004; Moschkovich, 2002, 2007). As noted by the National Clearinghouse for English Language Acquisition (NCELA) (2010), both nationally and in Connecticut, the rate of growth of ELLs in schools far exceeds that of other students (e.g., for graphic representations of CT trends see http://www.ncela.gwu.edu/files/uploads/20/Connecticut_G_0708.pdf and national trends see http://www.ncela.gwu.edu/files/uploads/9/growingLEP_0708.pdf). Further, there is evidence to suggest that ELLs are not performing as well as other students on mathematics assessments. For example, the 2009 *National Assessment of Educational Progress (*NAEP) results reported that nationally only 12% and in Connecticut only 9% of fourth-grade ELLs (as compared to 41% and 48%, respectively, of non-ELL fourth-grade students) were at or above proficient levels for mathematics. For eight grade, nationally only 6% and in Connecticut only 5% of students (as compared to 34% and 41%, respectively, of non-ELL eighth-grade students) were at or above proficient levels for mathematics (*National Center for Education Statistics* (NCES), 2009). Connections between language and mathematics achievement cannot be ignored.

Attention to language is critical for ELLs, but it is necessary for all students. Language is recognized as a vital mediator of learning (Vygotsky, 1978, 2002; Wertsch, 1991). It is a primary means for how we communicate about ideas and come to understand something new. Additionally, research demonstrates strong relationships between reading and math performance (e.g., Grimm, 2008), highlighting the importance of language development for academic success. Building mathematical meaning requires the ability to make sense of, and shift between, everyday language and the academic language – in particular, to language associated with the *mathematics register* – that is, "meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself)" (Halliday, 1978, p. 195).

There are two main purposes to this article. First, we aim to increase *awareness* of language in mathematics classes and features of language and language use that are important to attend to when developing students' mathematical proficiency, with specific focus on elementary grades mathematics classrooms. Second, we aim to offer teachers beginning steps for implementing a key strategy that has been shown to be effective in supporting language development in classrooms– the identification and development of language objectives (Echevarría, Vogt, & Short, 2007) – and which we used successfully in collaboration with a group of urban mathematics educators. To do this, we define language objectives, share specific examples and ideas for developing and using language objectives in elementary school math classes, and offer some ideas for how readers wishing to undertake this work might proceed.

Math ACCESS Project

Many of the ideas presented in this paper were developed as part of a professional development and research project, Math ACCESS (Academic Content and Communication Equals Student Success). The first year of the project was a small-scale pilot year (2007-2008) that involved four teachers (grades 4, 5, and 9). The second year of the project (2008-2009) was supported by the *Teacher Quality Partnership Grant* program from the Connecticut State Department of Higher Education and involved 23 teachers (grades 4 -10) from four schools in the same urban school district. The Math ACCESS project had a unique focus – working with teachers of mathematics to understand the language demands of student participation in higher order thinking, and developing strategies and lessons to address these demands and support student language development for mathematics. The project focused on three principles that research has found to be critical for the development of students' mathematical proficiency: appropriate and effective development of students' academic language, student engagement in mathematical practices of justification and collective argumentation, and access for all students to rigorous mathematics. For more detail on research literature related to each pillar, please see Staples & Truxaw, 2010a.

In both years of the project, there were two main components – summer professional development and academic year collaborative work. During the academic year, teachers and university personnel collaborated to develop, implement, and debrief higher-order thinking (HOT) lessons that used pedagogical strategies related to each of the three pillars. (The set of HOT lessons produced over the two years are archived at http://www.crme.uconn.edu/lessons/). For more information on the larger project, please see Truxaw & Staples, 2010.

Language Objectives

As part of the work with teachers, we introduced them to strategies associated with the SIOP® model (Echevarría, Vogt, & Short, 2007; 2010). SIOP®, formerly known as the Sheltered Instruction Observation Protocol, is an instructional approach that offers teachers a framework for planning and implementing high quality instruction for English language learners. The goal is not just to adapt lessons to remove linguistic barriers so students can engage content, but to purposefully *develop* students' language skills so they gain the linguist proficiencies and command of academic language needed to be successful in school and in subject areas like mathematics.

A key strategy to accomplish this goal is for teachers to identify and write *language objectives* for their lessons, and subsequently to design the lessons to include instructional experiences to help students meet the language objectives. Language objectives indicate what language and literacy skills students will develop as part of the activity. The SIOP® model describes several possible categories of language objectives including: key vocabulary, language functions, language skills, grammar or language structures, lesson tasks, and language learning strategies (see Table 1). Language objectives should be used in tandem with content objectives.

Some content and language objectives may overlap and some language objectives may not fit neatly into one category. The important idea is not that objectives need to be precisely categorized, but rather that a teacher's awareness of language demands, and *different types* of language demands, is useful and necessary for supporting students' academic growth in mathematics. This awareness should move beyond a focus on vocabulary to include attention to how language is used to *express mathematical ideas* and the development of the mathematics register (Moschkovich, 2002; Pimm, 1987; Schleppegrell, 2007). In other words, language objectives in math lessons should focus on language that is necessary to do and understand mathematics. Our particular focus was promoting and supporting higher order thinking in mathematics.

Table 1

Selected Categories of Language Objectives

Category	Descriptions	Example language objectives
Key Vocabulary	Developing familiarity with, and ability to, use words and phrases necessary to do and understand mathematics – for example, general academic, specific mathematical, and contextual words and phrases	Students will be able to (SWBAT) use appropriate content vocabulary in context: perimeter, area, length, width, dimensions
Language functions	Developing the ability to *use* language (beyond key vocabulary) to do and understand mathematics – for example, using language to describe, compare, explain, summarize, formulate questions, generalize, justify claims	SWBAT justify claims for which option is a better deal by offering statements and reasons (evidence). SWBAT explain how they calculated the area of the triangle using time sequence words
Language skills	Developing literacy and language *skills* necessary to do and understand mathematics – for example, learning to scan directions, find a main idea, restate a problem	SWBAT restate the question they need to answer. SWBAT restate, in their own words, the constraints of the problem.
Grammar or language structures	Developing ability to recognize and use grammar and language structures to do and understand mathematics – for example, pronoun usage, prepositions and prepositional phrases, adjectives or adverbs to indicate comparisons, extremes, etc.	SWBAT compare the values of fractional numbers by using comparative and superlative adjectives correctly, such as more than, less than, the largest, etc.

Writing Language Objectives

Writing language objectives for math classes is challenging, particularly at first. Like many aspects of teaching, however, over time it becomes easier, as experience is gained and as teachers become more aware of the nature of the language demands of doing mathematics. The ACCESS teachers overwhelmingly noted that they found this work difficult at first, but that they saw the payoff. Explicit inclusion of language objectives, including sharing them with the students, helped them to enhance their teaching and student learning.

In the following sections, we share a sample task and go through the process of developing language objectives. We first analyze the task, discussing its language demands. We then look at samples of associated student work to further explore language issues and hone in on areas where attention to language would be beneficial. In the final section, we offer sample language objectives for possible lessons related to the identified issues.

Identifying Language Demands

One approach to identifying language demands is to carefully review student tasks, and when possible, student work. Figure 1 shows a sample item for the Connecticut Mastery Test (CMT) for grade four. We used this prompt as a pre- and post-assessment for the Math ACCESS project. The work samples that follow were administered in fall 2008 (samples C, D and E) and spring 2009 (samples A and D). The one adaptation we made was to add the sentence "Explain how you know that you spent the least" to part *b* for the spring administration.

Hot Dog Buns

You estimate that you'll need 40 buns for a class picnic.

Hot dog buns are sold in packages of 8 and packages of 12.

- The package of 8 costs $1.00
- The package of 12 costs $1.20

a. Show three different ways you could buy packages to get at least 40 buns.

b. Which packages would you buy if you wanted to spend the least money? Show or explain how you arrived at your answer.

c. Which packages would you buy if you wanted exactly 40 buns?

Figure 1. Fourth-grade CMT sample item from State Dept. of Education, 2009.

This task required that students work through multiple tasks and constraints, including mathematical, contextual, and linguistic challenges. We highlight four of the language demands, corresponding to the categories of language objectives outlined above.

Key vocabulary. Students might be unfamiliar with "everyday" words and phrases (e.g., *package*, *purchase*, and *estimate*) that could impact their understanding of, and how they respond to, the problem.

Language functions. Students need to compare different solutions and justify their selection. This is required in part *b* and, to some extent, part *c* (though not of part *a*). Comparing and justifying are types of higher order mathematical thinking that require command of particular ways of expressing ideas.

Language skills. The students' general ability to read and interpret the prompt influences their mathematical work, both their understanding of the problem and what they think is necessary. In this prompt, students need not only to attend to the requirements of a specific part of the problem (*a*, *b*, or *c*), but also, to refer back to the overall constraints of the problem and/or to other parts of the problem. For example, figuring how to "spend the least money" (in part *b*), requires that students refer to the overall guidelines of the problem (you'll need 40 buns, buns are sold in package of 8 and 12, etc.) and to their work in part *a*.

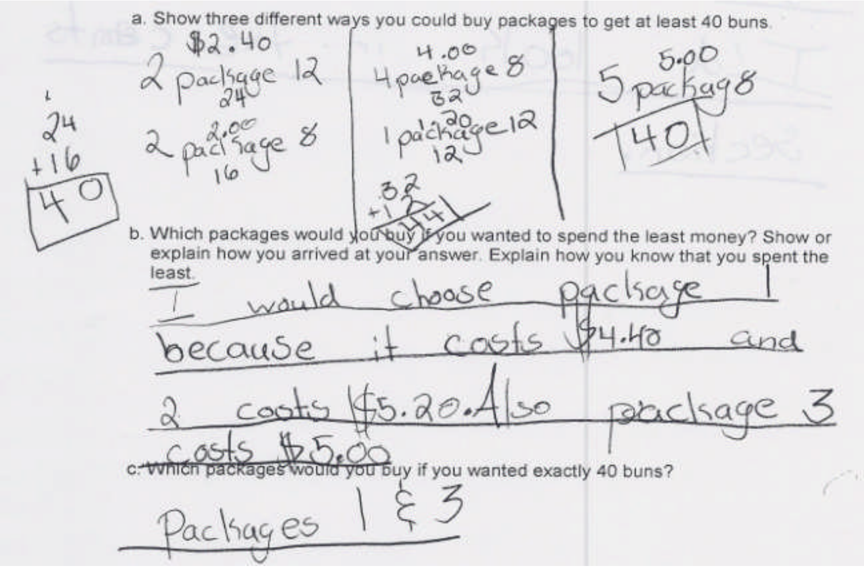
Grammar or language structures. The language challenges include grammar structures and phrases that are germane to the mathematical work students are expected to engage. For example, phrases such as "*at* least" (in part *a*) and "*the* least" (in part *b*) may be familiar, but may be misinterpreted or not understood, leading to very different mathematical work. Similarly, "at least" (in part *a*) and "exactly" (in part *c*) are quantifiers (adverbs), which shape students' mathematical work and define what counts as a permissible solution.

Teachers, alert to these language demands, might design into lessons components that actively address these aspects of the task so students can both engage in the desired mathematical activity in that lesson and learn how to be successful when faced with similar demands in the future. A review of samples of students' work related to this task makes even more apparent the nature of the language demands and some of the specific issues.

Student Work Samples – Language Demands

As we review samples of student work for the task shown in Figure 1, we suggest that the reader examine the student work samples to identify possible language issues (evidence from both understandings and misunderstandings) prior to reading our "take" on them. Following each student work sample, we provide our perceptions of language issues involved and suggest language objectives that might be pursued to address the issues. Our goal is to demonstrate that much can be learned from looking carefully at student work and that teachers can strategically target the issues they note and develop ways to move students forward linguistically and mathematically.

<u>*Student A*. Student A completed all three parts of the problem as shown in Figure 2.</u>



a. Show three different ways you could buy packages to get at least 40 buns.

$2.40
2 package 12
24
2.00
2 package 8
16

24
+16
40

4.00
4 package 8
32
1.20
1 package 12
12
32
+12
44

5.00
5 packag8
40

b. Which packages would you buy if you wanted to spend the least money? Show or explain how you arrived at your answer. Explain how you know that you spent the least.

I would choose package 1 because it costs $4.40 and 2 costs $5.20. Also package 3 costs $5.00

c. Which packages would you buy if you wanted exactly 40 buns?

Packages 1 & 3

Figure 2. Student A work sample.

Analysis of student A's work. Student A's mathematics and ability to appropriately interpret what the problem is asking seems fairly strong. In part *a*, this student demonstrates satisfactory understanding of the requirements of the problem, including using the identified sizes of packages (8 and 12) and interpreting "at least 40 buns" as 40 or more (rather than misinterpreting it to mean exactly 40). Student A's work is organized and labeled sufficiently to allow interpretation of its appropriateness and accuracy, though one could argue that it could be even more clearly labeled (e.g., labeling 40, 44, and 40 as total number of buns for each "way").

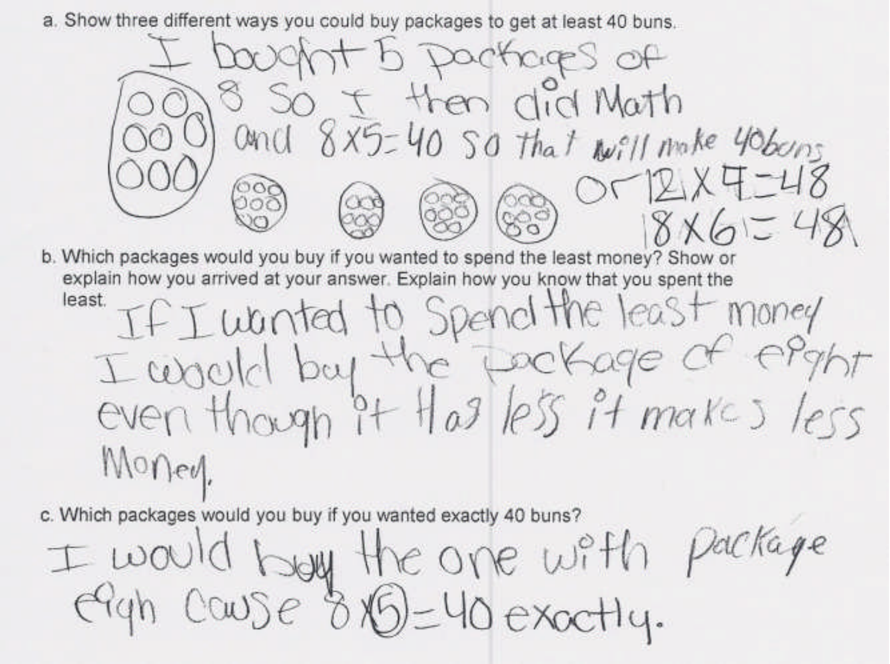
For part *b*, student A demonstrates a reasonable claim about which packages to choose to spend the least money. The student however says, "I would choose package 1" – using the word "package" to refer to the first of the three ways shown in part *a* – perhaps because of the phrasing of part *b*, which begins "Which packages would you buy…". The student also supports his/her claim with a reason – essentially the other two options cost more, though this could be more explicit. Part *b* is a good example of a mathematical response where the student answers the question and then supports her/his response with evidence (e.g., providing comparison prices), thus *justifying* his/her selection.

For part *c*, assuming that “Packages 1 & 3” refers back to part *a* of the problem (consistent with the use of the word package in part *b*), the student identifies the two ways of making exactly 40. Although student A shows some confusion about use of the word package, overall, this student’s responses show the ability to unpack the language of the problem sufficiently to figure out what math is involved. Further, this student is able to do the necessary math and to communicate it sufficiently well.

Language objectives to support student A. After reviewing student A’s work, we might suggest a language objective related to *key vocabulary* that focuses on specific, everyday words and phrases, for example, focusing on the meaning of the word “package” in the context of this problem, or how one can label work to reference it clearly.

Moving beyond vocabulary, and looking at student A’s justification in part *b* as nearly exemplary for a 4th-grade class, we might suggest an objective for the class, anticipating that others students might find part *b* rather challenging. In such a lesson, this student’s current work might serve as an example for others; and a language frame could be used as support to help others as they learn to justify responses (“I would choose ___ because ___.”). This category of objective is *language functions* related to further enhancing verbal and written justification.

Student B. Student B completed all three parts of the problem as shown in Figure 3.



a. Show three different ways you could buy packages to get at least 40 buns.

I bought 5 packages of 8 So I then did Math and 8x5=40 so that will make 40buns or 12x4=48 8x6=48

b. Which packages would you buy if you wanted to spend the least money? Show or explain how you arrived at your answer. Explain how you know that you spent the least.

If I wanted to Spend the least money I would buy the Package of eight even though it Has less it makes less Money.

c. Which packages would you buy if you wanted exactly 40 buns?

I would buy the one with package eigh cause 8x⑤=40 exactly.

Figure 3. Student B work sample.

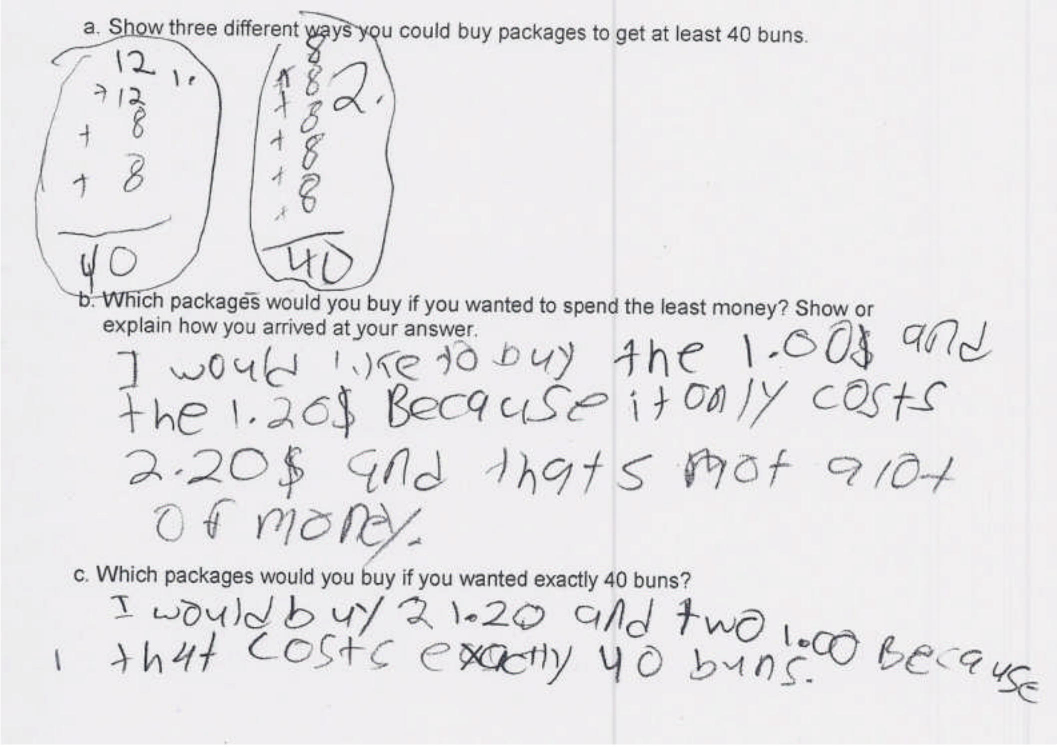
Analysis of student B's work. For parts *a* and *c*, student B's mathematics and ability to appropriately interpret the problem seem fairly strong. As with student A, we could think of extending this student's work to include additional necessary labels, for example, indicating that 12 x 4 was meant to represent 4 packages of 12 buns each, as the student had done with his first set of packages (five 8-bun packages). Note that it is not clear whether the student understands that s/he can *combine* 8-bun and 12-bun packages when making the purchase.

For part *b* this student offers a response that is recognizable as a justification. The student has a claim ("I would buy the package of eight") and offers the beginnings of supporting evidence or reasoning ("even though it has less it makes less money"). Thus the student understands the general *form* of producing a justification, offering a claim and a reason. However, the argument is not fully articulated or not correct. Student B's justification seems to focus on the cost of one package ($1 is less than $1.20), but should instead focus on the total cost of 40 (or more) buns. The mathematical error, then, is that the student is comparing the wrong quantity/unit. But this may not only be a question of mathematical understanding. Student B also notes that the package of eight has fewer buns ("even though it has less") than the package of 12. This suggests that this student is reasoning about how many buns are purchased, and not just the cost, which means there's some indication of proportional reasoning or thinking about cost relative to received goods. Consequently, it cannot be determined from this work sample whether the student's solution is limited because of language proficiency, and s/he is having trouble articulating ideas, or mathematical proficiency, where the student doesn't yet understand why it would not be appropriate to only compare the cost of one package of each when deciding the purchase. This would need to be further explored.

Language objectives to support student B. After reviewing student B's work, we would suggest language objectives related to *language functions* focusing on comparisons and writing justifications. To focus on comparisons, a teacher might pose questions about what is being compared in part *b* or what it means to spend the least money, as well as what in the prompt indicated to them that they should be looking at the total cost. The teacher could also pose the following question: "You have packages of 8. You need 24 buns, so you buy 3. You have packages of 12. You need 24 buns, so you buy 2. Which costs less?" and listen carefully for student responses to gauge mathematical and language proficiency.

To move from comparison toward justification, a language frame similar to the one mentioned in student A's analysis might prove useful – "I think ____costs the least because ____." If student A and B were in the same class, having them work together to compare and develop justifications may be of benefit to both of them. Students could also discuss in pairs what they are supposed to try to show in part *b*. This conversation likely would prompt a focus on exactly what is being compared (total cost or cost of the two types of packages).

Student C. Student C completed all three parts of the problem as shown in Figure 4.



a. Show three different ways you could buy packages to get at least 40 buns.

1.
12
+12
+ 8
+ 8
40

2.
8
+8
+8
+8
+8
40

b. Which packages would you buy if you wanted to spend the least money? Show or explain how you arrived at your answer.

I would like to buy the 1.00$ and the 1.20$ Because it only costs 2.20$ and thats not alot of money.

c. Which packages would you buy if you wanted exactly 40 buns?

I would buy 2 1.20 and two 1.00 because that costs exactly 40 buns.

Figure 4. Student C work sample.

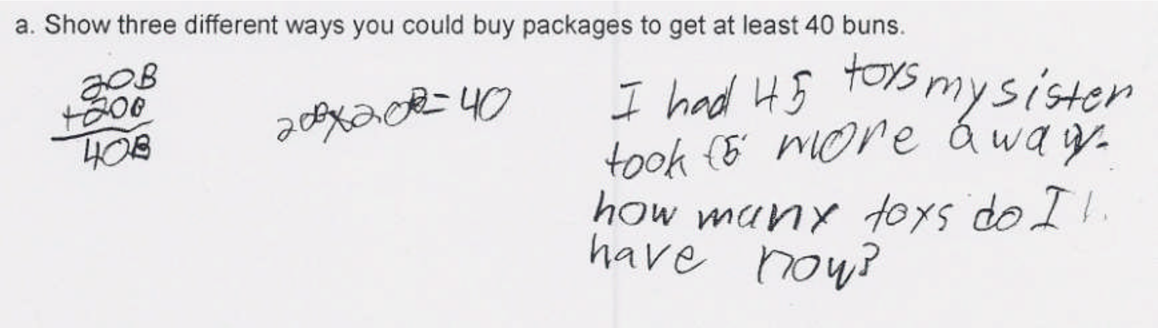
Analyzing student C's work. Part *a* above shows two "ways" of making "at least 40" using the identified sizes of packages (8 and 12). Note that both ways equal 40 exactly and there is no third way, suggesting that the student may have interpreted "at least 40 buns" to mean *exactly* 40 or may not have been sure if "at least 40" could include numbers greater than or less than 40. The student's scrap paper (not displayed) shows these two "ways" and one other that had been erased (perhaps one with a sum greater than or less than 40). Although we can't be certain from this work, checking for understanding of "at least" is warranted.

The work in part *b* is difficult to interpret. The student seems to indicate that it is not necessary to spend the *least* amount of money because it is not very much money to begin with. It is not clear why s/he picks one package of 8 and one package of 12. A conjecture is that s/he is not making connections between this question and other parts of the problem (e.g., part *a*). The student's interpretation brings up a potential critique of the wording of the problem itself, which was apparent in student A's work as well. The phrasing "Which packages…" seemed hard to interpret, whereas a phrase such as "Which of the ways…" or "Which of the combinations of packages…" might attune students more appropriately to what they are to compare.

In parts *b* and *c*, the student shows that s/he can put an argument together – both parts include a claim followed by "because…" However, there are some gaps between the questions asked and the arguments presented and some mismatches between claims and the evidence used to support them. Some conjectures follow: For part *b*, perhaps s/he interpreted "the least" to mean "a little" or a low price ("… thats not alot of money"). Another conjecture is that the student needs to understand that the question in part b is *implicitly* asking – if you wanted to spend the least amount of money *when you buy these 40 buns that you need to get* (it is not explicit in the problem). Interestingly, in part *c*, s/he gives an answer for what packages to buy to get 40 and to the least amount of money (part *b*), suggesting that language issues may be confounding some of this student's ability to demonstrate her/his mathematical competence.

Language objectives to support student C. For this student, language objectives that focus on *grammar or language structures* may be useful – for example, recognizing the difference between phrases "*at* least" and "*the* least" in this problem. Additionally, this student may benefit from objectives related to *language skills* that focus on reading and interpreting the problem and connecting one part of the problem with other parts of the problem. Finally, as with students A and B, this student may benefit from explicit attention to *language functions* related to comparisons and justifications. Likewise, similar to student B, student C may benefit from explicit work on interpreting the prompts and what is being asked.

Students D and E. Figures 5 and 6 show part *a* of the problem for students D and E, respectively. Note: Neither of these two students showed work for part *b* of the problem – student D left it blank and student E stated, "I don't understand."



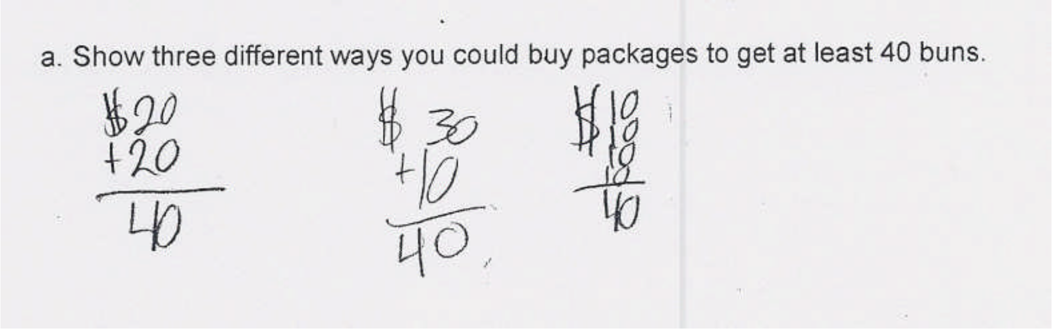
a. Show three different ways you could buy packages to get at least 40 buns.

20B
+20B
40B

20B×20B=40

I had 45 toys my sister took 5 more away. how many toys do I have now?

Figure 5. Student D: work sample for part *a*.



a. Show three different ways you could buy packages to get at least 40 buns.

$20
+20
40

$30
+10
40

$10
10
10
10
40

Figure 6. Student E: work sample, part *a*.

Analysis of students D's and E's work. Responses by students D and E suggest that there is confusion about how they interpreted "three different ways" in the context of this problem. Student D shows three "ways," namely, unique problems that had 40 as the solution. This student shows an addition problem with a sum of 40 that includes "B" labels (presumably buns), but that does not use the identified sizes of packages (8 and 12); a multiplication problem with an (incorrect) product of 40 that includes "B" labels (again, presumably buns) and that does not use the identified package sizes; and an unrelated subtraction word problem with a result of 40 ("I had 45 toys my sister took 5 more away how many toys do I have now?"). Student E shows three "ways," which in this case are unique sums to produce 40. Unlike student D, this student labels values as dollars and does not use the identified package sizes. For both of these students, it is clear that there are issues related to how they read, understood, and interpreted the problem. If one deletes some words from the prompt, we see the students possibly keyed in on the words that they did understand: Show three different ways ~~you could buy packages~~ to get ~~at least~~ 40 ~~buns~~. The students have understood that they need three different ways to produce something related to 40, but the other key phrases of the prompt do not seem to be understood.

Language objectives to support students D and E. For these students, productive language objectives could relate to *language skills* that focus on reading and interpreting the problem. For issues related to reading and interpreting the problem, we suggest that teachers consider using literacy strategies that they use in their teaching already *within their math classes*. Additionally, teachers may need to talk explicitly about the meaning of phrases such as "three different ways" – how phrases can mean different things in different problems.

Writing Corresponding Language Objectives

Table 2 provides examples of language issues discussed, the related category of language objective, and written sample language objective that might be used to focus a lesson and support the students. For example, one of the areas for learning noted in the above was that students might not fully understand the term *package* or perhaps what it means to say that one package is cheaper than another. An area to work with students is helping them think in different "units"—individual buns or packages of buns. When we say one of the packages is cheaper, what do we mean? Two follow up questions include: "Which package costs less?" and "Which package is the better deal?" This would get students understanding that each package contains a different number of hot dog buns, and that, overall, the buns (*each individual bun*) is cheaper when you buy a package of 12, even though you overall spend more money.

Table 2

Translating Issues from Student Work to Language Objectives

Language Issues	Categories of Language Objectives	Examples of Language Objectives
(Students A-E) Misunderstanding "everyday" language in the context of the problem.	Key Vocabulary	• SWBAT provide explanations for the following words: package, purchase, estimate

Language Issues	Categories of Language Objectives	Examples of Language Objectives
(Students D & E) Misunderstanding math language in the context of the problem	Key vocabulary	- SWBAT provide explanations for use of key phrases within the context of the problem. Language frame: "Show three ways means _____."
(Students B-E) Misunderstanding of how components of the problem relate to the larger problem	Language skills	- SWBAT to read and interpret components of the problem and identify when one component requires them to look for information elsewhere in the problem.
(Students D & E) Misunderstanding of main ideas of the problem and its constraints	Language skills	- SWBAT to restate the main ideas of the problem (and its sub-problems) in their own words. SWBAT identify and list the constraints of the problem.
(Students A, B, C) Articulation of math reasoning, comparison, and justification	Language functions	- SWBAT compare costs of different combinations of packages. - SWBAT to write a justification for how they know the least money was spent. Language frame: "I think __ costs the least because ____."
(Student C) Misunderstanding of "at least" in context of the problem	Grammar or language structures	- SWBAT to recognize and explain the difference between phrases "at least" and "the least" and provide appropriate examples of each.

For specific strategies to support teaching towards the language objectives, we suggest one of the SIOP texts (e.g., Echevarría, Vogt, & Short, 2007, 2010) that offer a wealth of strategies and activities that can be used for a range of purposes. We also document strategies used by the ACCESS teachers elsewhere (e.g., Staples & Truxaw, 2010b; Truxaw & Staples, 2010), and many are apparent in their lesson plans that are archived (see weblink above). Many of the strategies that are part of the SIOP® model for developing language are already familiar to teachers, particularly those in elementary grades who teach reading and language arts. For example, using sentence frames, word walls, showing models of student responses, and doing think-pair-share activities are all valuable strategies for developing language, whether it be vocabulary, command of grammar, the ability to justify or engage in other important mathematical practices, or to participate in general language-based activities required in classrooms such as reading, writing, speaking, and listening.

Discussion

The careful examination of even one CMT prompt, with student work samples, raises many issues related to math and language and the nature of support and learning students must experience in order to advance their language and math proficiency in tandem. Being successful in math requires command of particular language features and these must be explicitly taught in classrooms. All students are *math language learners,* and they require support for developing this proficiency in conjunction with their mathematical understandings. The task of learning math and math language is greater for those students who are English language learners as well, as they are working from a different foundation. Teachers' awareness of the role of language in mathematics classrooms can be invaluable in attuning the teacher to components of instruction and for including the requisite support to help students develop the skills they need to be successful in math.

To move forward, in addition to some of the readings already suggested, we have found that *collaboration* is the key for developing teachers' capacity to support their students in developing their mathematical academic language. Teachers can look at tasks together and analyze them for their language demands. Similarly, teachers can review student work samples and identify issues that would be productive to build into lessons, as well as other aspects of students' work that indicate areas for attention and growth. Through this process, teachers' awareness of the role of language in mathematics can be increased and teachers can generate possible approaches for addressing the identified issues. Our work with teachers has extended over two years and we feel like we have made good progress, but there is much more to learn. Each mathematical topic seems to bring some new issues that require our collective attention and the application of old strategies as well as the generation of some new ones. Over time, that tool kit builds, and those "language lenses" are sharpened, all in the service of supporting students' success in mathematics.

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C^2MT – Connecting Connecticut's Mathematics Teachers

Matching Expression

By Georgina Rivera

Middle School Teacher – Bristol, Connecticut

This lesson was developed to address the following goals:

- To assess students' ability to match an algebraic expression with the word phrase
- To assess student's knowledge of the vocabulary associated with the four basic operations

The lesson correlates to the following Connecticut Mastery Test and Common Core State Standards:

- **CMT Correlation:** Grade 7 Strand 23D and E
- **CCSS Correlation:** 6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.

It is offered here to assist 7th-grade teachers seeking to address these goals and standards.

Lesson Plan: Matching Expressions

Materials:

- Expressions and word phrases sheet
- Colored 8 ½ x 11 paper
- Scissors
- Glue Sticks

Activity:

Draw a table on the board that has four columns and above each column write the symbol for the four basic operations (see below). Ask the students to share words that mean the same as each of the operations and write them in the appropriate column. Once you have generated and listed all the responses either leave the table on the board or have them copy it onto white lined paper.

+	–	x	÷

Distribute the materials and tell students that they are going to match each algebraic expression to a word phrase. Tell students to fold the colored paper into sixteenths as shown below and cut out all of the expressions and word phrases on the Expressions and Word Phrases Sheet. Students will glue the algebraic expression to the left side of the colored paper and the matched word phrase next to it. They will need to use both sides of the colored paper to fit all the expressions and phrases. Once they have matched all the algebraic expressions with a word phrase, check their matches.

x + 7	**the sum of x and 7**

Discussion:
Ask students to explain the strategies they used to match the expressions and the word phrases.

Extension:
Ask students to write their own word phrases and the matching algebraic expressions on white lined paper and share them with the class. Challenge their peers to write an algebraic expression based on a word phrase they wrote.

Match the Expression Student Activity Sheet

$11 - y$	$\frac{7}{x}$	$7x$
$x - 7$	$y - 11$	$\frac{y}{11}$
$3y - 7$	$7x + 3$	$11 - y$
$x + 7$	$y + 11$	$7 - x$
$11y$	$x + 11$	$11y - 7$
7 minus a number	*The sum of x and 7*	*The product of 7 and x*
The quotient of 7 and a number	*7 less than a number*	*11 more than x*
The product of 7 and x plus 3	*The difference of 11 and y*	*11 times a number*
A number divided by 11	*11 more than y*	*A number decreased by 11*
11 take away y	*7 fewer than 3 times y*	*The product of 11 and a number decreased by 7*

The Connecticut Mathematical Journal
CAPT Corner

This month's contribution to CAPT Corner was submitted by Sarah Champagne, Patrick Hickey, and Eric Steinfeld, teachers at Bristol Eastern High School. They developed this problem to address the following Common Core State Standards and CAPT objectives:

- **Common Core State Standards Correlation –**

  Apply geometric concepts in modeling situations

  G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

  Find arc lengths and areas of sectors of circles

  G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

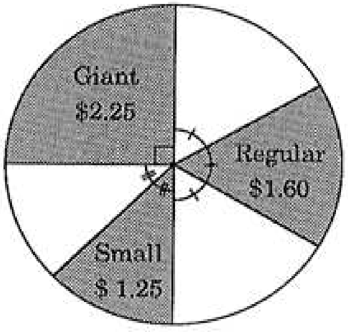
- **CAPT Correlation –**

  3.1b(2) Create logical arguments to solve problems and determine geometric relationships.

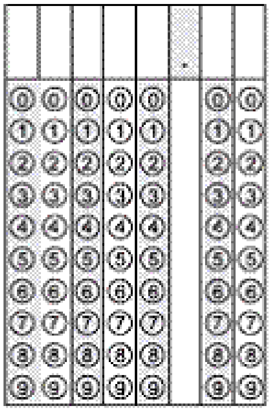
  3.1a(2) (2) Use geometric properties to solve problems in two and three dimensions.

Ms. Champagne's Pizzeria

1. Ms. Champagne's Pizzeria sells sauce-less cheese pizza by the slice according to the diagram below. Which slice gives the best deal (the most pizza for the dollar)? Show your work or explain your reasoning.



2. Ms. Champagne decides that her pizzeria needs to sell a stuffed crust pizza. She decides to buy Polly-O String Cheese and stuff the string cheeses into the crust. The diameter of a large pizza is 16 inches. Each string cheese link is 5 inches long. How many string cheese links will Ms. Champagne need to stuff the crust of a large pizza? Use the grid to record your answer.



Solutions:

1. This problem is rich since the student can approach the solution from different angles. Here are two possible solution routes.

   Method A: The student can compare the cost of the whole pizza by recognizing how many slices make up the whole pizza. Four Giant slices make a whole pie and would cost 4x$2.25 = $9.00. Six Regular slices make a whole pie and would cost 6x$1.60 = $9.60. Eight Small slices make a whole pie and would cost 8x$1.25 = $10.00. Thus the Giant slice would be the best buy.

   Method B: A student can also find the amount of pizza per dollar for each slice to compare the cost of the pizza. Since the Giant slice is one-fourth of the pizza, therefore ¼ of a pizza ÷ $2.25 = 0.111 pizzas/$ (rounded to the nearest thousandth). The Regular slice is one-sixth of the pizza, therefore 1/6 of a pizza ÷ $1.60 = 0.104 pizzas/$ (rounded to the nearest thousandth). Finally, the Small slice is one-eighth of the pizza, therefore 1/8 of a pizza ÷ $1.25 = 0.1 pizzas/$ (rounded to the nearest thousandth). Thus the Giant slice is still the cheapest.

2. The solution to this problem will depend on how the student interprets the problem. The exact length of string cheese needed is calculated by finding the perimeter of the 16-inch pizza and dividing by the length of one string cheese. The results of these calculations approximating π using 3.14, we see that the diameter of one pizza 16 inches • 3.14 = 50.24 inches. When we divide this by the length of one string cheese, 5 inches, we see that we need 10.048 string cheeses. The student may decide to give the answer as 10 since there is so little remaining to complete the circumference. That would be the logical answer. A student could give the more exact answer of 10.048 string cheeses, but that would seem somewhat impractical in this case. Finally, if students have been schooled in problems of this type and they were making only one pizza, they might say you have to have 11 string cheeses to completely fill the crust. Thus this problem becomes an excellent source of discussion for the class.

A Letter from the Editor

As you can see, this issue includes two articles based on work being done at the University of Connecticut. We appreciate their work and encourage other researchers and practitioners in the state to consider reporting their work in the Connecticut Mathematics Journal. It is a perfect forum for letting our colleagues around the state know what we are doing in the classroom.

It should also be noted that Bristol Public Schools provided both the CAPT Corner and C^2MT contributions. I would encourage other school districts to consider sending in activities and problems their teachers have developed to receive the recognition they highly deserve.

I look forward to receiving more contributions!

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